A SCN-TLM Model for the Analysis of Ferrite Media

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***Abstract*—The modeling of ferrite media using the symmetrical condensed node (SCN) of transmission lines matrix (TLM) method and current sources is developed. The proposed approach allows to study the interaction between electromagnetic (EM) waves and gyromagnetic media. The obtained results are in good agreement with the analytical ones.**

***Index Terms*—Ferrite media, time-domain electromagnetics, TLM method.**

I. Introduction

THE analytical study of electromagnetic waves interaction with ferrites is always a difficult task. Numerical techniques are often efficient tools for the analysis of such problems. The transmission line matrix (TLM) method has successfully dealt with linear isotropic dispersive media [1], [2], but it was seldom used for the direct analysis of gyromagnetic media [3]–[5].

In this paper, we develop a novel TLM model, using the symmetrical condensed node (SCN) and current sources, allowing a time domain study of electromagnetic (EM) waves propagation in ferrites media. The new formulation models the anisotropic and dispersive properties of ferrite media by adding current sources in supplementary stubs included in the standard SCN. The validity of this approach is proven by the study of the reflection and transmission of a plane wave normally incident on a magnetized ferrite wall.

II. Proposed Model

Ferrites are characterized in the frequency domain by a permeability tensor [$\mu(\omega)$]. The inverse Fourier transform of this tensor gives the susceptibility functions $\chi(t)$ and $\kappa(t)$ which, in the time domain, describe the anisotropic and dispersive properties of ferrites media and couple the components of the EM wave propagating in this medium [6]. In a ferrite medium with a static biasing magnetic field parallel to the z-axis, the EM field components, supposed constant in the time interval Δt in the 3-D lattice $(i\Delta x,\, j\Delta y,\, k\Delta z)$, are written at the instant $(n+1)\Delta t$ [6]

$$
{}_{n+1}\begin{pmatrix} H_x \\ H_y \\ H_z \end{pmatrix} = \begin{pmatrix} 1 & C_{mxy} & 0 \\ -C_{myx} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} {}_nH_x - \alpha_{mx} \cdot {}_nH_x + \beta_{mx} \\ \quad \cdot \left({}_n\psi_{mx} - \dfrac{\Delta t}{\mu_0}\, {}_{n+1/2}(\nabla \times E)_x \right) \\ {}_nH_y - \alpha_{my} \cdot {}_nH_y + \beta_{my} \\ \quad \cdot \left({}_n\psi_{my} - \dfrac{\Delta t}{\mu_0}\, {}_{n+1/2}(\nabla \times E)_y \right) \\ {}_nH_z - \alpha_{mz} \cdot {}_nH_z + \beta_{mz} \\ \quad \cdot \left({}_n\psi_{mz} - \dfrac{\Delta t}{\mu_0}\, {}_{n+1/2}(\nabla \times E)_z \right) \end{pmatrix} \tag{1}
$$

where

$$
C_{mxy} = \kappa^0_{xy}/(1+\chi^0_{yy}) \quad \text{and} \quad C_{myx} = \kappa^0_{yx}/(1+\chi^0_{xx}) \tag{2}
$$

are the magnetic coupling coefficients and

$$
\begin{pmatrix} {}_n\psi_{mx} \\ {}_n\psi_{my} \\ {}_n\psi_{mz} \end{pmatrix} = \begin{pmatrix} \sum_m^{n-1} \Delta\chi^m_{xx} \cdot {}_{n-m}H_x - \sum_m^{n-1} \Delta\kappa^m_{xy} \cdot {}_{n-m}H_y \\ \sum_m^{n-1} \Delta\chi^m_{yy} \cdot {}_{n-m}H_y + \sum_m^{n-1} \Delta\kappa^m_{yx} \cdot {}_{n-m}H_x \\ \sum_m^{n-1} \Delta\chi^m_{zz} \cdot {}_{n-m}H_z \end{pmatrix} \tag{3}
$$

are the discrete convolutions at the time $n\Delta t$ and are computed recursively. χ^m and κ^m are the generalized susceptibility [6]. The other parameters are defined by the following equations:

$$
\begin{pmatrix} \alpha_{mx} \\ \alpha_{my} \\ \alpha_{mz} \end{pmatrix} = \begin{pmatrix} (\chi^{0^2}_{xx} + \chi^0_{xx} + \kappa^{0^2}_{xy})/((1+\chi^0_{xx})^2 + \kappa^{0^2}_{xy}) \\ (\chi^{0^2}_{yy} + \chi^0_{yy} + \kappa^{0^2}_{yx})/((1+\chi^0_{yy})^2 + \kappa^{0^2}_{yx}) \\ \chi^0_{zz}/(1+\chi^0_{zz}) \end{pmatrix} \tag{4}
$$

and

$$
\begin{pmatrix} \beta_{mx} \\ \beta_{my} \\ \beta_{mz} \end{pmatrix} = \begin{pmatrix} (1+\chi^0_{xx})/((1+\chi^0_{xx})^2 + \kappa^{0^2}_{xy}) \\ (1+\chi^0_{yy})/((1+\chi^0_{yy})^2 + \kappa^{0^2}_{yx}) \\ 1/(1+\chi^0_{zz}) \end{pmatrix}. \tag{5}
$$

In order to develop a time domain approach modeling ferrite media by the TLM method, we should make use of the equations defined above. This consists first in adding to the standard (18 $\times$ 18) SCN three other supplementary stubs (19, 20, 21) in which current sources $(V_{six}, V_{siy}, V_{siz})$ are introduced to model the gyromagnetic properties. Then, we impose the condition of magnetic flux conservation in the node, we use the equivalence between voltages and EM field components, and finally we couple equivalent currents in the xoy plane. Therefore, we

Manuscript received September 9, 2002; revised November 28, 2002. This work was supported in part by the National Research Center of Morocco under Contract PROTARS II 2000-P12/23. The review of this letter was arranged by Associate Editor Dr. Rüdiger Vahldieck.

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Digital Object Identifier 10.1109/LMWC.2003.814105

find the expressions of the equivalent currents in the x, y, and z directions at the instant $(n+1)\Delta t$

$$ {}_{n+1}\begin{pmatrix} I_x \\ I_y \\ I_z \end{pmatrix} = \begin{pmatrix} 1 & C_{mxy} & 0 \\ -C_{myx} & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} {}_nI_x + \frac{1}{4+Z_{sx}}\left({}_{n+1}V_{six} + {}_nV_{six}\right) \\ -\frac{4}{4+Z_{sx}}\frac{\Delta t}{\mu_0}{}_{n+1/2}(\nabla \times E)_x \\ {}_nI_y + \frac{1}{4+Z_{sy}}\left({}_{n+1}V_{siy} + {}_nV_{siy}\right) \\ -\frac{4}{4+Z_{sy}}\frac{\Delta t}{\mu_0}{}_{n+1/2}(\nabla \times E)_y \\ {}_nI_z + \frac{1}{4+Z_{sz}}\left({}_{n+1}V_{siz} + {}_nV_{siz}\right) \\ -\frac{4}{4+Z_{sz}}\frac{\Delta t}{\mu_0}{}_{n+1/2}(\nabla \times E)_z \end{pmatrix}. \tag{6} $$

The comparison of (1) and (6) yields the expressions of the currents sources modeling gyromagnetic properties and the impedances of ferrite media

$$ \begin{pmatrix} {}_{n+1}V_{six} + {}_nV_{six} \\ {}_{n+1}V_{siy} + {}_nV_{siy} \\ {}_{n+1}V_{siz} + {}_nV_{siz} \end{pmatrix} = \begin{pmatrix} 4+Z_{sx} \\ 4+Z_{sy} \\ 4+Z_{sz} \end{pmatrix} \begin{pmatrix} -\alpha_{mx}\cdot{}_nI_x + \beta_{mx}\cdot{}_n\psi_{mx} \\ -\alpha_{my}\cdot{}_nI_y + \beta_{my}\cdot{}_n\psi_{my} \\ -\alpha_{mz}\cdot{}_nI_z + \beta_{mz}\cdot{}_n\psi_{mz} \end{pmatrix} \tag{7} $$

$$ \begin{pmatrix} Z_{sx} \\ Z_{sy} \\ Z_{sz} \end{pmatrix} = \begin{pmatrix} 4/\beta_{mx} - 4 \\ 4/\beta_{my} - 4 \\ 4/\beta_{mz} - 4 \end{pmatrix}. \tag{8} $$

These two former equations define the concepts necessary for the novel TLM approach based on the SCN and currents sources. Considering the magnetic flux conservation principle in the SCN, we obtain the expressions of the equivalent current, which are more convenient for the numerical treatment

$$ {}_{n+1}\begin{pmatrix} I_x \\ I_y \\ I_z \end{pmatrix} = \begin{pmatrix} \frac{2}{4+Z_{sx}} \\ \frac{2}{4+Z_{sy}} \\ \frac{2}{4+Z_{sz}} \end{pmatrix} \cdot \begin{pmatrix} {}_{n+1}(V_5^i + V_8^i - V_7^i - V_4^i + Z_{sx}V_{16}^i + 0.5V_{19}^i) \\ +C_{mxy}\left(\frac{4+Z_{sx}}{2}\right)\left({}_nI_y + {}_n\psi_{my}\right) \\ {}_{n+1}(V_2^i + V_{10}^i - V_9^i - V_6^i + Z_{sy}V_{17}^i + 0.5V_{20}^i) \\ -C_{myx}\left(\frac{4+Z_{sy}}{2}\right)\left({}_nI_x + {}_n\psi_{mx}\right) \\ {}_{n+1}(V_3^i + V_{12}^i - V_{11}^i - V_1^i + Z_{sz}V_{18}^i + 0.5V_{21}^i) \end{pmatrix}. \tag{9} $$

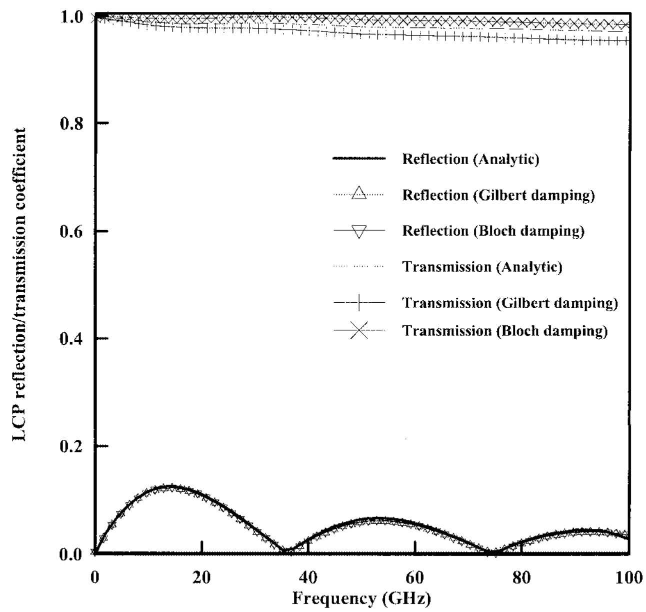

Fig. 1. LCP transmission and reflection coefficient magnitudes versus frequency.

Moreover, imposing the charge conservation principle to the node, we obtain the equivalent voltages

$$ {}_{n+1}\begin{pmatrix} V_x \\ V_y \\ V_z \end{pmatrix} = \begin{pmatrix} 0.5 \\ 0.5 \\ 0.5 \end{pmatrix} \begin{pmatrix} {}_{n+1}(V_1^i + V_2^i + V_{12}^i + V_9^i) \\ {}_{n+1}(V_3^i + V_4^i + V_{11}^i + V_8^i) \\ {}_{n+1}(V_5^i + V_6^i + V_{10}^i + V_7^i) \end{pmatrix}. \tag{10} $$

The scattering in the SCN modeling ferrite media is obtained according to the following procedure.

Equations (9) and (10) and those obtained from the continuity of the electric and magnetic fields expressed in terms of incident and reflected pulses allow to obtain scattering in the main twelve 1–12 lines of the SCN [7], the scattered pulses in the six stubs 13–18 can be obtained directly from

$$ \begin{pmatrix} V_{13} \\ V_{14} \\ V_{15} \\ I_{16} \\ I_{17} \\ I_{18} \end{pmatrix}^r = \begin{pmatrix} Vx \\ Vy \\ Vz \\ Ix \\ Iy \\ Iz \end{pmatrix} - \begin{pmatrix} V_{13} \\ V_{14} \\ V_{15} \\ I_{16} \\ I_{17} \\ I_{18} \end{pmatrix}^i. \tag{11} $$

Finally, the additional three stubs 19, 20, and 21 contribute solely to the incident pulses associated with current sources characterizing the gyromagnetic media.

III. Numerical Results

In order to validate the proposed model, the reflection and transmission of a Gaussian plane wave normally incident on an air-ferrite interface are investigated. The spatial TLM lattice considered is (1 × 1 × 800) Δl, the ferrite layer spans 50 Δl, with Δl is the mesh width taken to be $\Delta l = 75\ \mu$m. The propagation in the ferrite medium is described by Gilbert

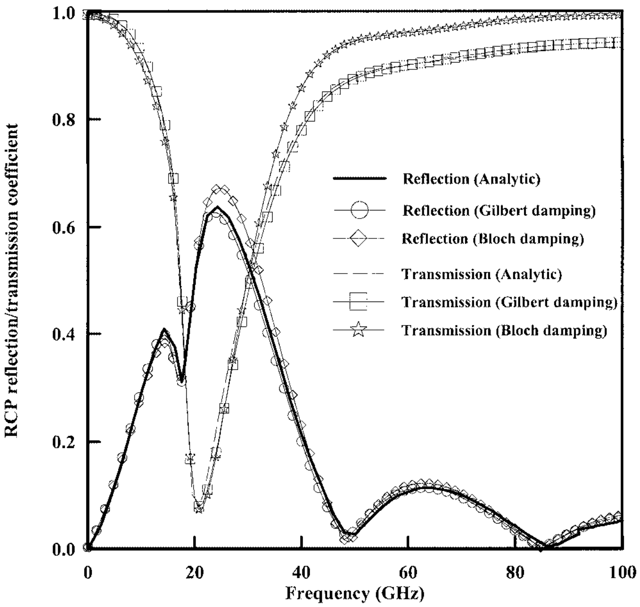

Fig. 2. RCP transmission and reflection coefficient magnitudes versus frequency.

model or Bloch model [3]. For the first model, the physical parameters characterizing the ferrite are: the precessional frequency $\omega_0 = 2\pi \times 20 \times 10^9$ rad/s, the magnetization frequency $\omega_m = 2\pi \times 10 \times 10^9$ rad/s and the damping constant $\alpha = 0.1$ [6]. When it comes to the second model, the characterizing parameters are $\omega_0 = 2\pi \times 19.8 \times 10^9$ rad/s, $\omega_m = 2\pi \times 10 \times 10^9$ rad/s and the transverse relaxation frequency $\nu_c = 2\pi \times 1.98 \times 10^9$ rad/s.

Figs. 1 and 2 show the reflection and transmission coefficients for left and right circularly polarizations (LCP, RCP). The agreement between our results and those obtained analytically is very good.

IV. Conclusion

A novel approach fort the modeling of ferrites in the time domain using the TLM method with the SCN and current sources is developed. The comparison of results emanating from this model with those obtained analytically reflects its validity and efficiency.

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